

# The Stochastic Buckling of Geometrically-Imperfect Hemispherical Shells Exhibits a Similar Size Effect as the Strength of Brittle Solids

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**Abstract:** Thin-walled shells and cylinders are widely used in engineering applications due to their excellent strength-to-weight ratio, but their structural stability can be compromised by randomly-distributed geometric imperfections introduced during fabrication. Such imperfections, even if small relative to other characteristic dimensions, can drastically reduce the load required to buckle the shells from the value predicted for perfectly shaped shells by linear buckling theory. While the sensitivity of shell buckling to imperfection geometry is well recognized, the probabilistic nature of these imperfections has received less attention. This seminar presents the results of two studies that fill that gap for two types of imperfections; those in the form of continuous deviations from the perfect geometry along the whole surface, and those in the form of localized dimples. First we examined the statistical distribution of the pressure required to buckle imperfect hemispherical shells with radius  $R$  and thickness  $t$ , by focusing on how the shell's slenderness  $R/t$  and the normalized correlation length of the random field representing the imperfection surface influence the mean and variability of the shell's buckling resistance. In a similar study an analytical model was derived for the buckling pressure distributions associated with localized dimples.



For the first type, which is the focus of this talk, we represent the geometric imperfections as a zero-mean homogeneous Gaussian random field applied on a hemispherical shell. Utilizing a stochastic finite element approach, Monte Carlo simulations are conducted for varying  $R/t$  and normalized correlation lengths. The computational results indicate the

presence of a “statistical size effect” and establish that  $R/t$  plays a previously unknown dual role, affecting both the deterministic features of local nonlinear deformation and the stochastic characteristics of buckling resistance. It is not surprising that equation derived for the effect of size on buckling resistance has the same form as a previously derived size effect equation for the strength of solids containing a random distribution of microcracks. This research provides a fundamental framework for reliability-based design of large-scale thin-walled structures, by providing insights into safety margins and scaling laws for hemispherical shells with random geometrical imperfections.

#### References:

- Z. Bhaizhikova, R. Ballarini and J.L. Le, “ Uncovering the Dual Role of Dimensionless Radius in Buckling of Spherical Shells with Random Geometric Imperfections,” PNAS, Vol. 121, No. 16, April 16, 2024, DOI 10.1073/pnas.2322415121.
- Z. Baizhikova, U.K. Ubamanyu, F. Derveni, R. Ballarini, P.M. Reis and J.L. Le, “A Probabilistic Buckling Model for Hemispherical Shells with Non-Interacting Localized Defects,” Journal of the Mechanics and Physics of Solids, pre-proof available at <https://doi.org/10.1016/j.jmps.2025.106468>